ON THE OPTIMIZATION OF A VIBRATION TRANSPORTER PROCESS

(OB OPTIMIZATSII PROTSESSA VIBROTRANSPORTIROVKI)

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V. A. TROITSKII
(Leningrad)

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Results are presented concerning a computation of the optimum regimes of vibration transport for conveyors with a horizontal driving mechanism. It is assumed that the conveying plane performs vibrations in a direction maing an angle with it or performs independent vibrations in the horizontal and vertical directions. Regimes of motion which are not accompanied by tossing up the transported particles are investigated.

The simplest problem of such a kind was studied in [1]. It was considered therein that the horizontal conveying plane performs vibrations in the horizontal direction. This problem is used as an example of the application of the general methods of investigation described herein.

1. Formulation of the problem. Vibration conveyors with a horizontal conveying plane are considered here. At first it is assumed that this plane vibrates with a given period T_0 along the ξ-axis which makes an angle β with the plane. The appropriate computational diagram is shown in Fig. 1. Also shown is the particle B whose motion is studied henceforth.

The particle B moves along the plane A under specific laws $\xi(t)$ of the motion of the plane. Those motions of the plane will be determined, which impart the maximum relative displacement to the particle per period. These laws are sought among all possible motions of the plane in the given period T_0.

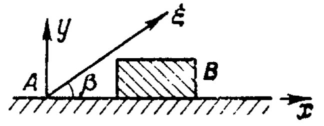

Fig. 1.

The equation of motion of a particle B without tossing up can be represented as [1,2]

$$m\,(\ddot{x} + \ddot{\xi}\cos\beta) = -\,fm\,(g + \ddot{\xi}\sin\beta)\,\mathrm{sign}\,\dot{x} \tag{1.1}$$

Here m is the particle mass; x its displacement along the plane; ξ the displacement of the plane; f the coefficient of friction; and g the acceleration of gravity.

Let us introduce the notation $\dot{x} = z$ and $\ddot{\xi} = u$. Then (1.1) is rewritten as

$$g^{\pm} = \dot{z}^{\pm} + u^{\pm}\cos\beta \pm f\,(g + u^{\pm}\sin\beta) = 0 \tag{1.2}$$

The upper symbol (+) and the upper sing in front of the last term correspond to the case $\vartheta = z > 0$. The lower symbol (-) and sign should be taken for $\vartheta = z < 0$.

The constraint

$$|\,u^{\pm}\,| \leqslant U^* \tag{1.3}$$

is henceforth imposed on the acceleration $\ddot{\xi} = u$ of the plane A.

If it reflects the demand that the particle not be separated from the plane then

$$U^*\sin\beta \leqslant g$$

Upon compliance with the inequality

$$U_1^* \leqslant u \leqslant U_2^* \qquad \left(U_1^* = -\frac{fg}{\cos\beta + f\sin\beta}, \quad U_2^* = \frac{fg}{\cos\beta - f\sin\beta}\right) \tag{1.4}$$

prolonged stoppage of the particle on the plane can occur. Hence, $z = 0$ and the relation

$$g^0 = \dot{z}^0 = 0 \tag{1.5}$$

should be used in place of equation (1.2).

The stoppage of the particle is denoted by the symbol "0".

Henceforth only steady-state periodic motions of the particles are considered. The dependences

$$\varphi_z = z\,(T) - z\,(t_0) = 0, \qquad \varphi_T = T - t_0 - T_\theta = 0 \tag{1.6}$$

are satisfied for such motions.

The condition of periodicity of the function $\xi(t)$ leads to the equality

$$\varphi_u^* = \int_{t_0}^{T} u(t)\, dt = 0 \tag{1.7}$$

The problem of optimizing the vibration transport process can be formulated as follows.

To find among the continuous functions $z(t)$ and the piecewise-continuous accelerations $u(t)$, which satisfy equation (1.2) and inequality (1.3) or equation (1.5) and inequality (1.4) in an interval of prescribed length $t_0 \leqslant t \leqslant T$ and relations (1.6) and (1.7) at its end points, those which are associated with the minimum value of the functional

$$I = -\int_{t_0}^{T} z(t)\, dt \tag{1.8}$$

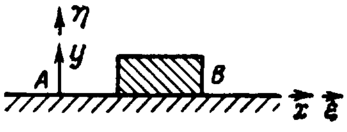

Fig. 2.

This will be called here the first problem of optimizing the operating regimes of vibration converyors.

The second problem corresponds to the diagram shown in Fig. 2. The conveying plane A performs independent vibrations in the horizontal (ξ) and (η) vertical directions. Among all the periodic functions $\xi(t)$ and $\eta(t)$ with a given period T_0 it is necessary to selct those which impart the maximum displacement to the particle within the period T_0.

The equation of motion of the particle B without tossing up can be represented in this case as

$$g^{\pm} = z^{\pm} + u_{\xi}^{\pm} \pm f(g + u_{\eta}^{\pm}) = 0 \tag{1.9}$$

Here $u_\xi = \ddot{\xi}$ and $u_\eta = \ddot{\eta}$, and the selection rule of the symbols and the sign in front of the last term agrees with that described above. The constraints

$$|u_{\xi}^{\pm}| \leqslant U_{\xi}^*, \qquad |u_{\eta}^{\pm}| \leqslant U_{\eta}^* \leqslant g \tag{1.10}$$

are imposed on the accelerations $\ddot{\xi}$ and $\ddot{\eta}$.

Upon compliance with the inequality

$$|u_{\xi}| \leqslant f(g + u_{\eta}) \tag{1.11}$$

prolonged stoppages of the particle on the plane can again occur.

Equation (1.5) is correct for them.

The condition of periodicity of the function $z(t)$ has the form (1.6). The second equality of (1.6) also remains in force. The periodicity of the functions $\xi(t)$ and $\eta(t)$ is expressed by the relations

$$\varphi_\xi^* = \int_{t_0}^{T} u_\xi(t)\,dt = 0, \qquad \varphi_\eta^* = \int_{t_0}^{T} u_\eta(t)\,dt = 0 \tag{1.12}$$

The second of the problems of optimizing the vibration transport process which are studied here is formulated as follows.

To find among the continuous functions $z(t)$ and the piecewise-continuous accelerations $u_\xi(t)$ and $u_\eta(t)$, which satisfy equation (1.9) and inequalities (1.10) or equation (1.5), inequality (1.11) and the first inequality of (1.10) in an interval $t_0 \leqslant t \leqslant T$ of given length $T - t_0 = T_0$ and relations (1.6) and (1.12) at its end points, those which are associated with a minimum value to the functional (1.8).

Closed domains of admissible changes in the parameters u, u_ξ and u_η are defined by inequalities (1.3) and (1.4) or (1.10) and (1.11). The transition to open domains is accomplished by the introduction of the additional parameters v, v_ξ and v_η and the construction of the auxiliary dependences

$$\begin{gathered}\Psi^\pm = (u^\pm)^2 + (v^\pm)^2 - (U^*)^2 = 0 \\ \Psi^0 = [(u^0 - U_1^*)(U_2^* - u^0) - (v^0)^2] = 0\end{gathered} \tag{1.13}$$

for inequalities (1.3) and (1.4) or the relations

$$\begin{gathered}\Psi_\xi^\pm = (u_\xi^\pm)^2 + (v_\xi^\pm)^2 - (U_\xi^*)^2 = 0, \quad \Psi_\eta^\pm = (u_\eta^\pm)^2 + (v_\eta^\pm)^2 - (U_\eta^*)^2 = 0 \\ \Psi_\xi^0 = [u_\xi^0 + f(g + u_\eta^0)]\,[f(g + u_\eta^0) - u_\xi^0] - (v_\xi^0)^2 = 0 \\ \Psi_\eta^0 = (u_\eta^0)^2 + (v_\eta^0)^2 - (U_\eta^*)^2 = 0\end{gathered} \tag{1.14}$$

for inequalities (1.10) and (1.11). By replacing the demand for compliance with inequalities (1.3) and (1.4) or (1.10) and (1.11) by these equations (1.13) or (1.14) in the formulations described above, we reduce them to the form which was considered in [1].

The difference in these formulations is that in the problems studied here there are two breaks in the continuity of the right-hand sides of the equations of motion. The equations and relations corresponding to this case are obtained from the appropriate replacements of the symbols established in [1]. They are presented below for the specific problems of optimum vibration transport solved here.

2. Construction of the optimum regime. *First problem.* Following the results of [1], let us form the functions H and φ

$$H^{\pm} = z^{\pm} + \lambda^{\pm}[\mp fg - u^{\pm}(\cos\beta \pm f\sin\beta)] + \rho u^{\pm} + \mu^{\pm}[(u^{\pm})^2 + (v^{\pm})^2 - (U^*)^2] \qquad (2.1)$$

$$H^0 = \rho u^0 + \mu^0 [(u^0 - U_1^*)(U_2^* - u^0) - (v^0)^2] \qquad (2.2)$$

$$\varphi = \rho_z [z(T) - z(t_0)] + \rho_T (T - t_0 - T_0) \qquad (2.3)$$

Using these, let us construct the equations

$$\dot{\lambda}^{\pm} = -1, \qquad \rho + 2\mu^{\pm}u^{\pm} - \lambda^{\pm}(\cos\beta \pm f\sin\beta) = 0, \qquad 2\mu^{\pm}v^{\pm} = 0 \qquad (2.4)$$

$$\dot{\lambda}^0 = 0, \qquad \rho + \mu^0(2u^0 - U_1^* - U_2^*) = 0, \qquad 2\mu^0 v^0 = 0 \qquad (2.5)$$

and the terminal conditions

$$\lambda(t_0) = \lambda(T) = \rho_z, \qquad (H)_{t_0} = (H)_T = \rho_T \qquad (2.6)$$

The Erdmann-Weierstrass conditions are represented by the equalities

$$\begin{aligned}
&\lambda^-(t^*) - \lambda^+(t^*) = 0, && (H^-)_{t^*} - (H^+)_{t^*} = 0 \\
&\lambda^{\pm}(t') - \lambda^{\mp}(t') + \nu_{\pm} = 0, && (H^{\pm})_{t'} - (H^{\mp})_{t'} = 0 \\
&\lambda^{\pm}(t') - \lambda^0(t') + \nu_{\pm}' = 0, && (H^{\pm})_{t'} - (H^0)_{t'} = 0 \\
&\lambda^0(t'') - \lambda^{\pm}(t'') + \nu_{\pm}'' = 0, && (H^0)_{t''} - (H^{\pm})_{t''} = 0
\end{aligned} \qquad (2.7)$$

The first pair is satisfied at the point $t = t^*$ of the break in the continuity of the parameters u and v. The rest correspond to times of the different discontinuities in the right-hand sides of the equations of motion [1]. Besides these equations and relations for the optimum regimes, the inequalities

$$\eta^{\pm}u^{\pm} \geqslant \eta^{\pm}U^{\pm}, \qquad \eta^0 u^0 \geqslant \eta^0 U^0 \qquad (2.8)$$

in which $u^{\pm}$ and u^0 correspond to the optimum regime and $U^{\pm}$ and U^0 are any admissible functions, are valid and the notation

$$\eta^{\pm} = \rho - \lambda^{\pm}(\cos\beta \pm f\sin\beta), \qquad \eta^0 = \rho \qquad (2.9)$$

has been introduced.

These inequalities are supplied by the necessary Weierstrass condition.

Let us note that the first integral

$$H = \rho_T \qquad (2.10)$$

formed such that conditions (2.6) and (2.7) would be satisfied, holds in this problem.

Relations (2.4) and (2.6) show that one of the following systems of dependences:

$$(1)\ \eta \neq 0,\ v = 0; (2)\ \eta = 0,\ v = 0; (3)\ \eta = 0,\ v = 0$$

can be satisfied in the optimum regimes.

By using the inequalities (2.8), we find for the first of them

$$u^{\pm} = \begin{cases} U^* & (\eta^{\pm} > 0), \\ -U^* & (\eta^{\pm} < 0), \end{cases} \qquad u^0 = \begin{cases} U_1^* & (\eta^0 = \rho < 0) \\ U_2^* & (\eta^0 = \rho > 0) \end{cases} \tag{2.11}$$

The periodicity condition (1.7) shows that not less than two changes in the sign of the parameter $u(t)$ should occur within the period in the optimum regime. Further analysis permits to verify that the functions $\lambda(t)$, which have discontinuities in the interval $t_0 \leqslant t \leqslant T$, correspond to this regime. Direct confirmation of the regimes of motion without prolonged stoppages of the particles by compliance with inequalities (2.8) affords the possibility of establishing that motions with prolonged stoppages correspond to optimum regimes. The first inequality of (2.8) is not satisfied for regimes without stoppages.

In constructing the optimum regime, let us assume, for definiteness, that $z(t_0) = 0$. Then a break in the continuity of the right-hand sides of the equations of motion and a discontinuity in the factor $\lambda(t)$ occur at $t = t_0$. In the sub-interval $t_0 < t < t_1$ following this point, the functions $\lambda(t)$ and $\eta(t)$ are continuous and represented by the relations

$$\begin{gathered} \lambda^+(t) = \rho_z - (t - t_0), \qquad \lambda^-(t) = \rho_z + (T_0 + t_0 - t) \\ \eta^+ = \rho - \rho_z(\cos\beta + f\sin\beta) + (\cos\beta + f\sin\beta)(t - t_0) \\ \eta^- = \rho - \rho_z(\cos\beta - f\sin\beta) + (\cos\beta - f\sin\beta)(t - t_0 - T_0) \end{gathered} \tag{2.12}$$

written down for both the cases $z > 0$ and $z < 0$.

Upon compliance with the inequality $\cos\beta - f\sin\beta > 0$, the function $\eta(t)$ increases together with t. Hence, $\eta(t_0 + 0) < 0$ and $u(t_0 + 0) = -U^*$. Then $z(t_0 + 0) > 0$ and it is necessary to assign the "plus" symbol to all the variables of the problem.

At a certain time $t = t_1^*$ the function $\eta^+(t)$ vanishes and then becomes positive. Hence, $u^+(t_1^* + 0) > 0$. In the sub-interval $t_1^* < t < t_1'$ the velocity $z(t)$ remains positive and becomes zero for $t = t_1'$. The lengths of the sub-intervals mentioned here are related *via* the dependence

$$\frac{t_1' - t_0}{t_1^* - t_0} = \frac{2U^* (\cos\beta + f \sin\beta)}{fg + U^* (\cos\beta + f \sin\beta)} \leqslant 2$$

Inequality

$$t_1' - t_1^* < t_1^* - t_0 \tag{2.13}$$

follows from it.

The assumption that the particle continues to move in the subinterval $t_1' < t < t_0 + T_0$ leads to a regime without particle stoppage. There are no optimum regimes among them. Hence, we consider that the particle B is fixed in this subinterval. Then relations (2.2) and (2.10) yield $\rho u^0 = \rho_T$ so that the parameter u^0 does not change sign. On the basis of dependences (2.11), we have $u^0 = U_1{}^*$ or $u^0 = U_2{}^*$. Periodicity condition (1.7) takes the form

$$-U^* (t_1^* - t_0) + U^* (t_1' - t_1^*) + u^0 (t_0 + T_0 - t_1') = 0$$

Hence, using inequality (2.13), we obtain

$$u^0 (t_0 + T_0 - t_1') = U^* (t_1^* - t_0) - U^* (t_1' - t_1^*) > 0$$

$$u^0 = U_2{}^* \tag{2.14}$$

Therefore, the optimum operating regime of a vibration conveyer is characterized by the following values of the acceleration of the conveying plane:

$$u(t) = \begin{cases} -U^* \ (t_0 < t < t_1^*), \\ U^* \quad (t_1^* < t < t_1'), \\ U_2{}^* \ (t_1' < t < t_0 + T_0), \end{cases} \quad \begin{aligned} t_1^* &= t_0 + \frac{U_2{}^*}{2U^*} \frac{fg^1 + U^* (\cos\beta + f\sin\beta)}{fg + U_2{}^* (\cos\beta + f\sin\beta)} T_0 \\ t_1' &= t_0 + \frac{U_2{}^* (\cos\beta + f\sin\beta)}{fg + U_2{}^* (\cos\beta + f\sin\beta)} T_0 \end{aligned} \tag{2.15}$$

The following expression:

$$-I = \frac{T_0{}^2 (\cos\beta + f\sin\beta)}{16 U^* \cos^2\beta} [(U^*)^2 (\cos\beta + f\sin\beta) - (fg)^2]$$

can be constructed for the displacement of the particles per period.

Shown in Fig. 3 are optimum functions $z(t)$ and $u(t)$ for the values

$$\beta = 30^\circ, \qquad U^* = 2fg, \qquad f = 0.5$$

3. Construction of the optimum regime. *Second problem.* Let us form the functions H and φ

$$H^{\pm} = z^{\pm} + \lambda^{\pm}[-u_{\xi}^{\pm} \mp f(g + u_{\eta}^{\pm})] + \rho_{\xi}u_{\xi}^{\pm} + \rho_{\eta}u_{\eta}^{\pm} + \mu_{\xi}^{\pm}[(u_{\xi}^{\pm})^2 + (v_{\xi}^{\pm})^2 - (U_{\xi}^{*})^2] + \mu_{\eta}^{\pm}[(u_{\eta}^{\pm})^2 + (v_{\eta}^{\pm})^2 - (U_{\eta}^{*})^2] \quad (3.1)$$

$$H^0 = \rho_{\xi}u_{\xi}^0 + \rho_{\eta}u_{\eta}^0 + \mu_{\eta}^0[(u_{\eta}^0)^2 + (v_{\eta}^0)^2 - (U_{\eta}^{*})^2] + \mu_{\xi}^0\{[u_{\xi}^0 + f(g + u_{\eta}^0)][f(g + u_{\eta}^0) - u_{\xi}^0] - (v_{\xi}^0)^2\} \quad (3.2)$$

$$\varphi = \rho_z[z(T) - z(t_0)] + \rho_T(T - t_0 - T_0) \quad (3.3)$$

Using them let us form the equations

$$\dot{\lambda}^{\pm} = -1, \qquad -\lambda^{\pm} + \rho_{\xi} + 2\mu_{\xi}^{\pm}u_{\xi}^{\pm} = 0, \qquad \mp\lambda^{\pm}f + \rho_{\eta} + 2\mu_{\eta}^{\pm}u_{\eta}^{\pm} = 0 \quad (3.4)$$
$$\mu_{\xi}^{\pm}v_{\xi}^{\pm} = 0, \qquad \mu_{\eta}^{\pm}v_{\eta}^{\pm} = 0$$

$$\dot{\lambda}^0 = 0, \qquad \rho_{\xi} - 2\mu_{\xi}^0u_{\xi}^0 = 0, \qquad \rho_{\eta} + 2f(g + u_{\eta}^0) + 2\mu_{\eta}^0u_{\eta}^0 = 0 \quad (3.5)$$
$$\mu_{\xi}^0v_{\xi}^0 = 0, \qquad \mu_{\eta}^0v_{\eta}^0 = 0$$

and terminal conditions (2.6). The Erdmann-Weierstrass conditions have the form (2.7). There is the first integral (2.10) in this problem also.

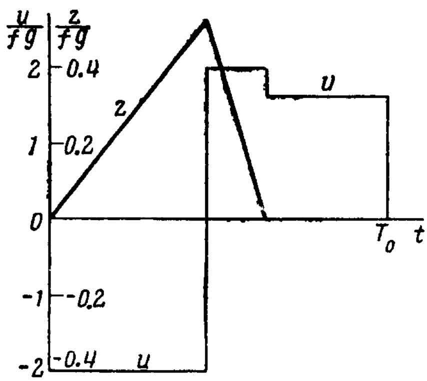

Fig. 3.

The necessary Weierstrass conditions for a strong minimum of the functional I adds the following inequalities to the relations listed above:

$$\eta_{\xi}^{\pm}u_{\xi}^{\pm} + \eta_{\eta}^{\pm}u_{\eta}^{\pm} \geqslant \eta_{\xi}^{\pm}U_{\xi}^{\pm} + \eta_{\eta}^{\pm}U_{\eta}^{\pm} \quad (3.6)$$
$$\eta_{\xi}^0u_{\xi}^0 + \eta_{\eta}^0u_{\eta}^0 \geqslant \eta_{\xi}^0U_{\xi}^0 + \eta_{\eta}^0U_{\eta}^0$$

The notation therein is

$$\eta_{\xi}^{\pm} = \rho_{\xi} - \lambda^{\pm}, \qquad \eta_{\xi}^0 = \rho_{\xi} \quad (3.7)$$
$$\eta_{\eta}^{\pm} = \rho_{\mu} \pm f\lambda^{\pm}, \quad \eta_{\eta}^0 = \rho_{\eta}$$

and u_{ξ} and u_{η} are understood to be the optimum laws of the change in acceleration, and U_{ξ} and U_{η} are any admissible functions.

The functions $U_{\xi} = u_{\xi}$ and $U_{\eta} = u_{\eta}$ are admissible. Putting $U_{\eta}^{\pm} = u_{\eta}^{\pm}$ in (3.6), we find $\eta_{\xi}^{\pm}u_{\xi}^{\pm} \geqslant \eta_{\xi}^{\pm}U_{\xi}^{\pm}$. The inequalities

$$\eta_{\eta}^{\pm}u_{\eta}^{\pm} \geqslant \eta_{\eta}^{\pm}U_{\eta}^{\pm}, \; \eta_{\xi}^0u_{\xi}^0 \geqslant \eta_{\xi}^0U_{\xi}^0, \; \eta^0u_{\eta}^0 \geqslant \eta_{\eta}^0U_{\eta}^0$$

are constructed in a similar manner. On this basis we obtain the following results:

$$u_\xi^{\pm} = \begin{cases} U_\xi^* (\eta_\xi^{\pm} > 0), \\ -U_\xi^* (\eta_\xi^{\pm} < 0), \end{cases} \qquad u_\eta^{\pm} = \begin{cases} U_\eta^* (\eta_\eta^{\pm} > 0) \\ -U_\eta^* (\eta_\eta^{\pm} < 0) \end{cases}$$

$$u_\xi^0 = \begin{cases} f(g + u_\eta^0)\,(\eta_\xi^0 > 0), \\ -f(g + u_\eta^0)\,(\eta_\xi < 0), \end{cases} \qquad u_\eta^0 = \begin{cases} U_\eta^*\,(\eta_\eta^0 > 0) \\ -U_\eta^*\,(\eta_\eta^0 < 0) \end{cases} \tag{3.8}$$

The factor $\lambda^{\pm}(t)$ has the form $\lambda = C - t$ during particle motion, where C is an integration constant. Then $\eta_\xi^{\pm}$ and $\eta_\eta^{\pm}$ can vanish at a finite number of points of the interval $t_0 \leqslant t \leqslant T$. The quantities η_ξ^0 and η_η^0 also differ from zero for particle stoppages at $\rho_\xi \neq 0$ and $\rho_\eta \neq 0$. Therefore, in this case u_ξ and u_η take on only the boundary values given by relations (3.8).

It is necessary to look for the optimum regimes in the second problem among the regimes to which particle motion with prolonged stoppages corresponds. A point $t = t'$ belonging to the interval $t_0 \leqslant t \leqslant T$ is certainly found in these regimes where $z(t') = 0$ and $z(t' + 0) > 0$. Otherwise, the regime ($u_\eta = 0$) found above would not be optimum.

Let us select the abscissa $t = t_0$ of the left-hand end of the interval $t_0 \leqslant t \leqslant T$ so that the equality $z(t_0) = 0$ and the inequality $z(t_0 + 0) > 0$ would be satisfied. Let us consider the subinterval $t_0 \leqslant t \leqslant t_1^*$. We have $\lambda^+ = \rho_z - t$ and $\eta_\xi^+ = \rho_\xi - \rho_z + t$ and $\eta_\eta^+ = \rho_\eta - f\rho_z + ft$ so that the functions η_ξ and η_η will be increasing functions of time.

Let us consider that $U_\xi^* > f(g + U_\eta^*)$. Then $\eta^+(t_0 + 0) < 0$ so that $u_\xi^+(t_0 + 0) = -U_\xi^* < 0$. In this case, constants ρ_ξ and ρ_z exist such that the equality $\eta^+(t_1^*) = 0$ is satisfied at a certain time $t = t_1^*$, where $\eta^+(t_1^* + 0) > 0$ and $u_\xi(t_1^* + 0) = U_\xi^* > 0$. Returning to equation (1.9), we find $\dot{z}(t_1^* + 0) < 0$. The relative velocity of the particle $z(t)$ decreases and vanishes for $t = t_1'$.

The velocity $z(t)$ is positive in the subinterval $t_0 < t < t_1'$. Hence, in this subinterval the function $\eta_\eta^{\pm}$ is continuous. Therefore, the acceleration u_η^+ takes on one of its boundary values $u_\eta^+ = \pm U_\eta^*$. The break in the continuity of u_η^+ can hold at the point $t = t_2^*$ at which $\eta_\eta^+(t_2^*) = 0$. If $t_1' - t_0 \geqslant 1/2\, T_0$, then such a point will certainly drop into the subinterval $t_0 < t < t_1'$ because otherwise the second periodicity condition (1.12) will be violated. Then $\eta_\eta^+(t_0 + 0) < 0$ and $u_\eta^+(t_0 + 0) = -U_\eta^* < 0$, where $u_\eta^+(t) = -U_\eta^*$ in the subinterval $t_0 < t < t_2^*$.

Let us consider the subinterval $t_1' < t < T$. Let us assume that the particle is fixed therein relative to the conveying plane. Then, on the basis of relations (3.2) and $H = \rho_T$, we will have $\rho_\xi u_\xi^0 + \rho_\eta u_\eta^0 = \rho_T$ since u_ξ^0 and u_η^0 are continuous and have the values (3.8) in this subinterval, where the values are determined by the signs of the constants ρ_ξ and ρ_η.

Hence, discontinuities in the acceleration $u_\xi(t)$ can occur at the points $t = t_0$, $t = t_1^*$ and $t = t_1'$; where $u_\xi(t_0 + 0) = -U_\xi^*$. Similarly, the vertical acceleration $u_\eta(t)$ can have discontinuities at the points $t = t_0$, $t = t_2^*$ and $t = t_1'$. It has the value $u_\eta = -U_\eta^*$ in the subinterval $t_0 < t < t_2^*$.

Let us study first the case $t_2^* > t_1^*$. Integrating equation (1.9), we obtain

$$z^+(t) = [U_\xi^* - f(g - U_\eta^*)](t - t_0) \qquad (t_0 \leqslant t \leqslant t_1^*)$$

$$z^+(t) = [-U_\xi^* - f(g - U_\eta^*)](t - t_0) + 2U_\xi^*(t_1^* - t_0) \qquad (t_1^* \leqslant t \leqslant t_2^*)$$

$$z^+(t) = -[U_\xi^* + f(g + U_\eta^*)](t - t_0) + 2fU_\eta^*(t_2^* - t_0) + 2U_\xi^*(t_1^* - t_0)$$
$$(t_2^* \leqslant t \leqslant t_1')$$

For $t = t_1'$ the last of these expressions yields

$$z^+(t_1') = -[U_\xi^* + f(g + U_\eta^*)](t_1' - t_0) + 2fU_\eta^*(t_2^* - t_0) + 2U_\xi^*(t_1^* - t_0) \quad (3.9)$$

Let us form the periodicity condition. Let us assume still that $t_1' - t_1^* > t_1^* - t_0$. Then the first condition of (1.12) becomes

$$-U_\xi^*(t_1^* - t_0) + U_\xi^*(t_1' - t_1^*) + f(g + u_\eta^0)(T - t_1') = 0$$

Similarly, let us consider that $t_2^* = t_0 + 1/2\, T_0$. Then the second equality of (1.12) will be satisfied identically, where the condition just formed is rewritten as

$$-U_\xi^*(t_1^* - t_0) + U_\xi^*(t_1' - t_1^*) + f(g + U_\eta^*)(T - t_1') = 0$$

It is easily converted to the form

$$-2U_\xi(t_1^* - t_0) + [U_\xi^* - f(g + U_\eta^*)](t_1' - t_0) + f(g + U_\eta^*)T_0 = 0 \quad (3.10)$$

The expressions

$$t_1' = t_0 + \frac{T_0}{2}\frac{g + 2U_\eta^*}{g + U_\eta^*}, \qquad t_1^* = t_0 + \frac{T_0}{4}\left[\frac{g + 2U_\eta^*}{g + U_\eta^*} + \frac{fg}{U_\xi^*}\right] \quad (3.11)$$

will be a solution of equations (3.9) and (3.10) for $t_2^* = t_0 + 1/2\, T_0$.

It is easy to see that for $U_\xi^* > f(g + U_\eta^*)$ the inequalities

$$^1/_4 T_0 < t_1^* - t_0 < ^1/_2 T_0, \quad t_1' - t_0 > ^1/_2 T_0 \tag{3.12}$$

which show that the constructed optimum regime exists, are satisfied for the quantities found.

Retaining the relation $t_2^* > t_1^*$ and assuming that $t_1' = t_2^* - 1/2\ T_0$ we arrive at negative values for $t_1^* - t_0$ and $t_2^* - t_0$. Such a regime cannot occur. Negative values are obtained for $t_1^* - t_0$ and $t_2^* - t_0$ when $t_2^* < t_1^*$ and $t_1' - t_2^* = 1/2\ T_0$ so that this regime also does not exist. Finally, for $t_2^* < t_1^*$ and $t_2^* - t_0 = 1/2\ T_0$ the lengths of the subintervals $t_1^* - t_0$ and $t_1' - t_0$ are determined by formulas (3.11) which, as inequalities (3.12) show, controdict the original assumptions. Such a regime may not exist.

Hence, the accelerations u_ξ and u_η are determined by the dependences

$$u_\xi = \begin{cases} -U_\xi^* & (t_0 < t < t_1^*), \\ U_\xi^* & (t_1^* < t < t_1'), \\ f(g + U_\eta^*) & (t_1' < t < t_0 + T_0), \end{cases} \qquad u_\eta = \begin{cases} -U_\eta^* & (t_0 < t < t_2^*) \\ U_\eta^* & (t_2^* < t < t_0 + T_0) \end{cases} \tag{3.13}$$

in the optimum operating regime of a vibration conveyer during independent vibrations of the conveying plane in the vertical and horizontal directions.

The quantities t_1^* and t_1' entering therein are given by (3.11) and $t_2^* = t_0 + 1/2\ T_0$.

The formula

$$-I = \frac{T_0^2}{16 U_\xi^*} \left\{ (U_\xi^*)^2 \left(1 + \frac{U_\xi^*}{g + U_\eta^*} \right)^2 - (fg)^2 + \frac{4f U_\xi^* (U_\eta^*)^2}{g + U_\eta^*} \right\} \tag{3.14}$$

can be obtained for the particle displacement in a period.

Now, let us consider three optimum operating regimes of a vibrating conveyer with a horizontal driving mechanism. Let us consider the acceleration U^* of the plane to be horizontal in the first regime, inclined at an angle of $\beta = 30^\circ$ to the plane in the second and that there are horizontal and vertical independent accelerations $U_\xi^* = U^* \cos\beta$ and $U_\eta^* = U^* \sin\beta$ in the third. Let us use the following values in the computation: $U^* = 3.6\ fg$ and $f = 0.5$. Then we will have $U_\xi^* = 3.12\ fg$ and $U_\eta^* = 1.8\ fg$.

The particle displacement per period in the first regime is $I = 0.208\ fgT_0^2$. In the second it has the value $I = 0.418\ fgT_0^2$. And, finally, it is given in the third by the equality $I = 0.550\ fgT_0^2$.

A comparison of these quantities affords the possibility of extablishing a significant increase in the displacement with the appearance of a vertical component of the acceleration. The effect of this component is particularly noticeable for an independent change in the vertical and horizontal accelerations.

Let us note that analogous problems were considered by Kopylov [3]. The optimum parameters of biharmonic regimes were determined in [4].

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